Factoring the Sum and Difference of Two Cubes

Here are 2 special properties that apply to adding or subtracting two perfect cubes:

\[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \quad a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]

Examples:

Factor each expression.

1) \[ x^3 - 8 \]
\[ (x - 2)(x^2 + 2x + 4) \]

2) \[ x^3 + 64 \]
\[ (x + 4)(x^2 - 4x + 16) \]

Practice:

Factor each expression.

1) \[ x^3 + 125 \]
\[ (x + 5)(x^2 - 5x + 25) \]

2) \[ x^3 - 27 \]
\[ (x - 3)(x^2 + 3x + 9) \]
The Factor Theorem

The Factor Theorem states the relationship between the linear factors of a polynomial expression and the zeros of the related polynomial function.

\[ x - r \] is a factor of the polynomial expression that defines the function \( P \) if and only if \( r \) is a solution of \( P(x) = 0 \), that is, if and only if \( P(r) = 0 \).

With the Factor Theorem, you can test for linear factors involving integers by using substitution.

Example:

Use substitution to determine whether \( x + 2 \) is a factor of \( f(x) = x^3 - 2x^2 - 5x + 6 \).

Determine what value for \( x \) makes \( x + 2 = 0 \).

Then find \( f(-2) \).

\[
\begin{align*}
\text{f}(-2) &= (-2)^3 - 2(-2)^2 - 5(-2) + 6 \\
&= 0
\end{align*}
\]

Because \( f(-2) = 0 \), the Factor Theorem states that \( x + 2 \) is a factor of \( x^3 - 2x^2 - 5x + 6 \).

Practice:

Use substitution to determine whether \( x - 1 \) is a factor of \( x^3 - x^2 - 5x - 3 \).

\[
\begin{align*}
\text{f}(1) &= 1^3 - 1^2 - 5(1) - 3 \\
&= 0
\end{align*}
\]

Not a factor